A Monopolistic Credit Rating Agency*

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Abstract

The paper analyses the demand for credit rating services of a continuum of firms. The firms differ in the probability of their investment's success which is private information. They can use the service of a monopolistic rating agency that sends an imperfect signal of their success probability to the capital market. The demand for rating services turns out to be not always monotonous in its price. If a rating agency exists, only rated firms obtain a credit. There can be oversupply or undersupply of rating services from a social planner's point of view.

Keywords: Credit Rating Agency, Monopoly, Oversupply, Perfect Bayesian Equilibrium

JEL classification: D42, D82, G21, L12, L15

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1 Introduction

In recent years more and more firms seek to finance their investments on the capital market and use the service of a rating agency.[1] The rating agency evaluates the firm's default risk for the credit it wants to obtain on the capital market. For this service the firm has to pay a fee. Although in reality rating agencies also earn some money from the subscribers of their publications, the major part of their profits stems from the fees the firms pay for their ratings.[2] Of course, the better the rating the smaller are the financing costs of a firm.

Although there is currently an intensive public discussion of the rating agencies' impact on the capital markets, there are only a few analytical contributions in the literature addressing this problem.[3] Nayar (1993) tries to model the demand for the services of a rating agency in order to discuss whether the rating of corporate securities in Malaysia should be mandatory or voluntary. He analyzes a partially separating equilibrium, but he does not derive the optimal strategies of the rating agency. Kuhner (2001) analyses rating agencies in times of enhanced systemic risks and models them as frontrunners in a Bayesian herding process. Since the debtor is not an active player in his model, he cannot derive the demand for rating services. Consequently, the agency's profit maximizing rating fee is also not determined. The same is true for Ramakrishnan and Thakor (1984) as well as Millon and Thakor (1985). They consider information producers in financial markets and refer explicitly to rating agencies as examples for such producers. They address the problem that the information producer might not have an incentive to put effort into the information production process.[4]

This paper focusses on the derivation of a demand for rating services, the strategic price setting behavior of a rating agency and its repercussions on the capital market. I assume a continuum of firms. They differ ex ante only in the success probability of a single investment project which is pri-

[1]See e.g. the article on the growing European rating market in *The Financial Times* from November 26, 1998, p. 23.

[2]See e.g. the article on Standard & Poor's and Moody's in *The Financial Times* from August 25, 1997, p. 12, where the author assesses that they earn 90 % of their revenues from these fees.

[3]See Fight (2001) for a good overview of the public debate and a critical survey of the rating industry.

[4]Recently, Mukhopadhyay (2000) analyses the price setting and the quality decision of a monopolistic rating agency with a single firm and a single investor. Therefore he cannot analyse the interdependence of the firms' rating decisions. In addition the demand for rating services is nearly always independent of the rating fee.

vate information. They have to finance their projects on the capital market where capital suppliers compete à la Bertrand. Thus, the interest rate factor coincides with the risk free interest rate factor corrected with the market's evaluation of the respective firm's success probability. The firms may demand a rating of their default risk from a monopolistic rating agency. If they do, the rating agency sends an imperfect signal to the capital market. The rating technology is such that the imperfect signal may be either"high" or "low". The probability that a rating results in a "high" signal is positively correlated to the success probability of the firm's investment project. The rating agency sets the price of its services and the firms decide whether they demand a rating or not. Afterwards the capital market responds to the rating and the high or low signal. Finally, financed firms pursue their projects under the conditions offered to them by the capital market.

It turns out that the demand for the rating service is not monotonous in the rating agency's price if the investment's pay-off is high relative to the risk-free interest rate factor. If non-rated firms are also funded by the capital market, raising the rating agency's price generates a higher demand for rating services. The rating agency, however, chooses the rating fee in such a way that only the rated firms are financed by the capital market. From a social welfare point of view this may lead to an oversupply of rating services. The rating agency may also oversupply in order to prevent an equilibrium without any demand.

In our setting firms cannot influence the expected profitability of their investment projects. Thus, our problem is a typical problem of adverse selection as introduced by Akerlof (1970). The rating is a signalling device, but, contrary to the majority of signalling models, the cost of the signal is not exogenously fixed.[5] It is strategically set by the rating agency.

Biglaiser (1993) is also an example where the cost of the signal is not exogenously fixed. In his model of costly bargaining between buyers, sellers and a middleman owners of high quality products signal their quality by selling it through a middleman. He can invest in a technology which enables him to determine the quality with some costs. Low quality owners sell directly to buyers. Welfare is improved by the middleman if the bargaining costs of sellers and buyers are not too high and if the discount factor is high enough. The latter describes the cost of the alternative signalling device for high quality sellers who may signal their quality also by waiting. In our framework the costs of using the signalling device of a rating agency is also crucial for

[5]See Spence (1973) and for other signalling models in finance with exogenous costs e.g. Leland and Pyle (1977), Miller and Rock (1985) and Milde and Riley (1988).

its welfare enhancing or deteriorating character.

Albano and Lizzeri (1997) and Lizzeri (1999) analyze a certification intermediary. Of course, a rating agency can be understood as such a certification intermediary. In their framework, however, the intermediary can observe the quality of the certified product perfectly, whereas here the rating technology which the rating agency can use in order to test the quality of the firms is imperfect. They analyze the information revelation strategy and the price setting strategy of the certification intermediary. Albano and Lizzeri (1997) focus, however, on moral hazard issues, because firms can decide on the quality of their products. Lizzeri (1999) deals with given product qualities, meaning that he considers an adverse selection problem as we do here. His perfect certification technology is, however, costless. Thus, social welfare is either not influenced by the intermediary's strategy or it is improved. Since the rating agency in our framework does not know more than that the signal is high or low, it does not make too much sense to address information revelation strategies in this context.

Melumad and Thoman (1990) analyze nonmandated auditing. Their monitor model has a remote resemblance to the rating model developed here. Their firms are either good and have a high probability to stay solvent or they are bad and have a low probability. If a firm hires a monitor, the firm publishes a statement whether it is good or bad. Then the monitor investigates and truthfully reports its findings to the public, but may fail to detect a false statement by a firm. Melumad and Thoman (1990) conclude that there are circumstances under which all firms are better off without monitors. The same is true in our framework, although, contrary to their analysis, we consider strategic price setting by the rating agency and we allow for firms whose investments are socially undesirable if their type were revealed.

In the next section we present our model. Then we show under which conditions firms obtain a credit if no rating agency exists. We derive the demand for rating services, the equilibria with a rating agency and we solve for the welfare maximizing supply of rating services. Finally we draw some conclusions.

2 The Model

We assume a continuum of risk neutral firms with their mass normalized to one. Each firm can pursue a single investment project. The projects of the firms are identical with respect to the pay-off in case of success and the necessary financial resources. The discounted pay-off is $x > 1$, if a firm succeeds, and zero otherwise. The necessary financial resources are normalized to one. The firms differ in their success probability p which is private information. The success probability p is uniformly distributed on the support $[0, 1]$. The firms have to finance their projects completely on the capital market. Firms maximize expected profits when they decide whether to demand the service of the rating agency or not.

The risk free interest rate factor is also normalized to one. The risk neutral lenders on the capital market can not observe p, but know its distribution. They compete in a Bertrand-like fashion for the financing of the projects, but can always use the outside option to lend their money risk-free. This means that $\rho_j r = 1$ must hold in equilibrium where r is the interest rate factor a firm has to pay if ρ_j is the lenders' belief of the firm's success probability, given that the firm is rated "low" $(j = l)$ or "high" $(j = h)$ or whether it is not rated $(j = n)$. Thus, a firm's interest rate factor r is given by:

$$r = \frac{1}{\rho_j}. \tag{1}$$

The rating agency has an imperfect rating technology that produces the signal "high" with probability p and "low" with probability $1 - p$. Thus, the higher the firm's success probability p the higher is the chance that the rating result will be "high" instead of "low". The rating agency truthfully reports the signal to the capital market. It has constant rating costs of c per rated firm. The rating agency sets a uniform rating price k in order to maximize its profits.

The sequence of the agents' decisions is the following:[6]

1. The rating agency sets the rating price k.
2. After having learnt their individual success probability, the firms decide simultaneously and non-cooperatively whether they demand a rating or not.
3. After having observed all the ratings, the lenders compete for the financing of the firms. The financed firms pursue their investment projects.

The firms and the lenders play a perfect Bayesian equilibrium. This means that they maximize their respective profits. The lenders form their beliefs according to Bayes' rule and these beliefs are correct in equilibrium. The monopolistic rating agency anticipates the respective perfect Bayesian equilibrium and sets its profit maximizing price accordingly.

[6]Note that in our framework it does not matter whether nature distributes the success probabilities in stage zero or between stage one and two.

3 The Market without the Rating Agency

The expected profit of a firm is $\pi(p) = \max\{p(x-r), 0\}$ without the option to demand rating services. If the obligations from the credit exceed the pay-off of the investment project, the lenders are not fully repayed. Given this limited liability, firms always invest, if they are able to attract the necessary financial resources on the capital market.

The lenders cannot distinguish between firms and form their expectation about the success probability of the firms according to the prior distribution: $\rho_n = \int_0^1 p dp = \frac{1}{2}$. Thus, competition among the lenders drives the interest rate factor r down to (see equation (1)) $r = 2$. The lenders, however, only give a credit at this interest rate factor, if the pay-off exceeds the interest rate factor, meaning $x > 2$. This proves the following Proposition:

Proposition 1 *If no rating agency exists and $x > 2$ all the firms can finance their projects at an interest rate factor of $r = 2$ at the capital market. If $1 < x \leq 2$ holds, no firm is financed.*

Thus, if the pay-off x is small relative to the risk free interest rate factor, then no firm is financed, even if their individual success probability were sufficient to satisfy the lenders.[7]

4 The Market with the Rating Agency

4.1 The Demand for Rating Services

When a rating agency offers its services, the firms can decide whether they want to get rated. Thus, a firm's expected profit is[8]

$$\pi(p) = \begin{cases} \max\left\{p\left(x - \frac{1}{\rho_n}\right), 0\right\} \text{ without a rating,} \\ p \max\left\{p\left(x - \frac{1}{\rho_h}\right), 0\right\} + (1-p)\max\left\{p\left(x - \frac{1}{\rho_l}\right), 0\right\} - k \\ \text{ with a rating.} \end{cases} \tag{2}$$

[7] This is the typical adverse selection result in models of asymmetric information without any signalling device. See Akerlof (1970).

[8] Note that the rating fee k has to be payed upfront. If we extended the limited liability concerning the credit obligations also to the payment of the rating fee, the results we obtain here would even be strengthened.

The firm has an incentive to demand a rating as long as its expected profit with a rating exceeds the expected profit without a rating for given beliefs of the lenders. Taking into account that in a perfect Bayesian equilibrium the firms' decisions must be consistent with the lenders' beliefs, one can derive the following lemma:[9]

Lemma 1 *In a perfect Bayesian equilibrium with a positive demand for rating services all firms with $p > \bar{p}$ demand a rating and all those with $p \leq \bar{p}$ do not obtain a rating. The threshold $\bar{p}$ is implicitly defined by (13).*

The firms with a high success probability have the strongest incentive to obtain a rating because they are highly undervalued without a rating, and they have the best chances to obtain a "high" rating.

The perfect Bayesian equilibria with a positive demand are further characterized in Table 1 where $\rho_n(\bar{p}) < \rho_l(\bar{p}) < \rho_h(\bar{p})$ are defined in (16), (15), and (14) in the Appendix. They refer to the capital market's belief of a firm's success probability, given that it is not rated, that it is rated "high" and that it is rated "low", when all the firms with $p \geq \bar{p}$ demand a rating.

Case	Restriction on Beliefs	Equivalent Restriction on $\bar{p}$
I	$x > 1/\rho_n(\bar{p})$	$\bar{p} > 2/x$
II	$1/\rho_l(\bar{p}) \leq x \leq 1/\rho_n(\bar{p})$	$\min\{2/x, 1\} \geq \bar{p} \geq \max\{(3-x)/(2x), 0\}$
III	$1/\rho_h(\bar{p}) \geq x < 1/\rho_l(\bar{p})$	$(3-x)/(2x) > \bar{p} \geq$ $\max\{(3-2x+\sqrt{3(3+4x-4x^2)})/(4x), 0\}$
IV	$x < 1/\rho_h(\bar{p})$	$\bar{p} < (3-2x+\sqrt{3(3+4x-4x^2)})/(4x)$

Table 1: Cases of Perfect Bayesian Equilibria with a Positive Demand

Since a firm is not financed, if $x < 1/\rho_j$, four cases can be distinguished according to the number of rated firms determined by $\bar{p}$. In Case I all the firms are financed. Only firms with very high success probabilities demand a rating. Thus, the average success probability of the unrated firms is still high enough that the capital market also finances their projects. This can only occur if $x > 2$, meaning that the investment's pay-off in case of success is

[9] All the proofs are relegated to the Appendix.

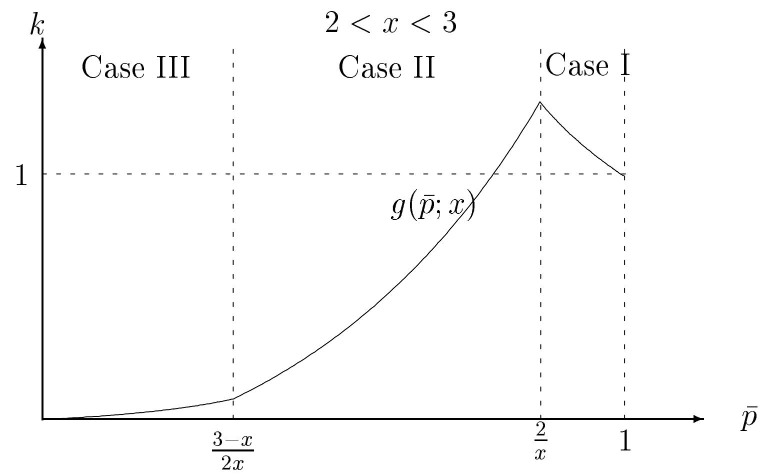

Figure 1: Marginal Willingness to Pay for a Rating

that high that the lenders would also finance all the firms, if no rating agency existed (see Proposition 1). In Case II more firms are rated. Thus, all the rated firms, no matter whether they received a "high" or a "low" rating, are financed, but the unrated firms are not financed anymore . In Case III only the "high" rated firms obtain a credit. The "low" rated and the unrated firms are not financed. In Case IV so many firms are rated that their average risk is too high. Neither the rated nor the unrated firms are financed. The latter situation can only occur if $x < 3/2$ meaning that the investment's pay-off in case of success is rather low.

The demand for rating services in a perfect Bayesian equilibrium with a positive demand coincides with $1 - \bar{p}$, where $\bar{p}$ is implicitly defined by (13) in the Appendix. Substituting (16), (15), and (14) in (13) and rearranging yields the function

$$g(\bar{p}; x) = \begin{cases} \frac{1}{2}\left(\frac{4+6\bar{p}+9\bar{p}^2-\bar{p}^3}{(1+2\bar{p})(1+\bar{p}+\bar{p}^2)}\right) \text{ in Case I,} \\ \bar{p}\left[x - \frac{3}{2}\left(\frac{2+\bar{p}+3\bar{p}^2}{(1+2\bar{p})(1+\bar{p}+\bar{p}^2)}\right)\right] \text{ in Case II,} \\ \bar{p}^2\left[x - \frac{3}{2}\left(\frac{1+\bar{p}}{1+\bar{p}+\bar{p}^2}\right)\right] \text{ in Case III,} \\ 0 \text{ in Case IV.} \end{cases} \tag{3}$$

It represents the rating fee k which ensures $\bar{p}$ as the relevant threshold, and, thus $1 - \bar{p}$ as the demand in the Bayesian equilibrium with a positive demand. Thus, it characterises the marginal willingness to pay for a rating, given a certain demand. It is depicted in Figure 1. The function $g(\bar{p}; x)$ is

k	Necessary condition for x	Financing
$0 \leq k < g\left(\frac{3-x}{2x}; x\right) = \frac{(x-3)^2(x-1)}{4(3+x^2)}$	$1 < x < 3$	only "high" rated firms (Case III)
$g\left(\max\left\{\frac{3-x}{2x}, 0\right\}; x\right) \leq k \leq \min\{x-1, 1\}$	$x > 1$	all the rated firms (Case II)
$1 < k \leq g\left(\frac{2}{x}; x\right) = \frac{2(x^3+3x^2+9x-2)}{x^3+6x^2+12x+16}$	$x > 2$	all the rated firms (Case II) or all the firms (Case I)

Table 2: The Perfect Bayesian Equilibrium(-a) with a Positive Demand Resulting from a Particular rating fee k

monotonously increasing in $\bar{p}$, if $x \leq 2$, and has a maximum in $\bar{p}$ at $\bar{p} = 2/x$, if $x > 2$. This proves the following Proposition:

Proposition 2 *For $\bar{p} < 2/x$ an increase in the rating fee k reduces the demand for rating services in a Bayesian equilibrium with a positive demand. If $\bar{p} > 2/x$ holds, increasing k, increases the demand for rating services.*

The marginal willingness to pay for a rating decreases in $\bar{p}$ for $\bar{p} > 2/x$ because the unrated firms also obtain a credit in this case. An increase of $\bar{p}$, meaning a decrease in the rating demand, results in a higher average success probability and, thus, in a lower interest rate factor not only for the rated, but also for the unrated firms and the latter effect more than compensates the former. From the analysis of $g(\bar{p}; x)$ the following results concerning the existence and uniqueness of the Bayesian equilibrium with a positive demand can be derived.

Lemma 2 *A perfect Bayesian equilibrium with a positive demand for the rating services always exists for a small but non-negative rating fee. If $1 < x \leq 2$ the perfect Bayesian equilibrium with a positive demand is unique. If $x > 2$, it is unique for low rating fees. For a sufficiently high rating fee (see Table 2) there are two perfect Bayesian equilibria with a positive demand.*

With $x > 2$ and a high enough rating fee two perfect Bayesian equilibria with a positive demand exist, one with a high demand for rating services and no financing of the unrated firms, and one with a low rating demand where all firms are financed. If $x \leq 2$ the latter equilibrium disappears, because no firm would have been financed without any rating (see Proposition 1). Thus, if some firms decide in favor of a rating, the average quality of the unrated firms is further reduced, compared to the case without any rating, and the capital market will even be less inclined to finance their projects.

It is obvious that a perfect Bayesian equilibrium where no firm gets rated always exists for any k, if the out-of-equilibrium beliefs about the success probability of rated firms are only pessimistic enough.[10] It seems reasonable, however, to restrict the out-of-equilibrium beliefs of the lenders. They should take into account that, if a firm with a certain success probability gets rated, those with an even higher success probability do have an even higher incentive to obtain a rating. Thus, in order to support a Bayesian equilibrium without any demand for the rating services we consider for the rest of the paper only those out-of-equilibrium beliefs that satisfy the following restriction:

Assumption 1 *The out-of-equilibrium beliefs of the lenders for an equilibrium where no firm gets rated are always characterized by a certain threshold $\tilde{p}$ for which they assume that all firms with $p \geq \tilde{p}$ are rated and all those with $p < \tilde{p}$ are not rated.*

Under this restriction we test the most pessimistic beliefs of the lenders that all the firms have obtained a rating. We summarize our results in Lemma 3.

Lemma 3 *A perfect Bayesian equilibrium in which no firm demands a rating exists, if $k > \min\{1/2, x - 3/2\}$.*

Note that there is no perfect Bayesian equilibrium where all firms are rated and $k > 0$ holds. Even if the lenders believe that the success probability of an unrated firm is close to zero, it does not pay for a firm with a high risk ($p \to 0$) to demand a rating. It could expect only a very small return on the investment, but would have to pay the rating fee k for sure.

[10] If the lenders believe, e.g., that the success probability of a rated firm is close to 0, then they would demand an almost infinite interest rate factor from such a firm. Thus, no firm can improve its profit by a rating and the equilibrium with no rating would exist independently of the rating agency's price.

4.2 The Rating Agency's Pricing Decision

Lemma 2 and Lemma 3 imply that the perfect Bayesian equilibrium that might be generated by a certain rating fee k may not be unique. Thus, we assume here that in case of multiple equilibria firms select one of the perfect Bayesian equilibria and the rating agency anticipates the firms' equilibrium selection.[11]

If there is a positive demand of rating services, the monopolistic rating agency's profit is given by:

$$\Pi(\bar{p}; x, c) = \int_{\bar{p}}^{1} (k - c)dp = (k - c)(1 - \bar{p}) = (g(\bar{p}; x) - c)(1 - \bar{p}). \tag{4}$$

We assume that the rating agency chooses k independently of the produced signal. Otherwise the rating agency's reputation as a credible source of information for the capital market would presumably be damaged. The monopolistic rating agency maximizes its profit with respect to k. This is equivalent to the maximization of (4) with respect to $\bar{p}$. The Analysis of the rating agency's profit function yields Lemma 4 where $\underline{a}_1 = \underline{a}(2/x, x)$ with

$$\underline{a}(\bar{p}, x) = \left\{ \frac{c}{x} \middle| \frac{\partial \Pi(\bar{p}; x, c)}{\partial \bar{p}} = 0 \right\}. \tag{5}$$

Lemma 4 *The rating agency's profit function $\Pi(\bar{p}; x, c)$ has an interior maximum at $\bar{p} \in (\max\{(3 - x)/(2x), 0\}, \min\{2/x, 1\}]$ if $c/x \leq \min\{\underline{a}_1, 1 - 1/x\}$ where $\underline{a}_1$ is defined in (23). Otherwise it has either a single peak at $\bar{p} = 2/x$ if $x > 2$ or, if $x \leq 2$, is monotonously increasing in $\bar{p} \in [0, 1]$.*

The range from which $\bar{p}$ can be chosen is, however limited depending on the selected perfect Bayesian equilibrium in case of multiple perfect Bayesian equilibria. The different ranges are given in Table 3 where

$$\bar{p}_k = \{\bar{p} < 1 \,| g(\bar{p}; x) = 1\}, \tag{6}$$

$$\bar{p}_{01} = \left\{ \bar{p} \middle| g(\bar{p}; x) = \frac{1}{2} \right\} \text{ and } \bar{p}_{02} = \left\{ \bar{p} \middle| g(\bar{p}; x) = x - \frac{3}{2} \right\}. \tag{7}$$

If the firms select the perfect Bayesian equilibrium without any demand as soon as it exists, the rating agency can either prevent its existence by

[11] In addition we assume that firms select one type of equilibrium and do not switch, e.g., from an equilibrium where all the firms are financed to one where only the rated firms are financed if k changes for all k that generate multiple equilibria.

	Firms select PBE with $\bar{p} \leq 2/x$	Firms select PBE with $\bar{p} > 2/x$	Firms select PBE with $\bar{p} = 1$
range of $\bar{p}$ if $x > 2$	$\bar{p} \in [0, 2/x]$	$\bar{p} \in [0, \bar{p}_k]$ $\wedge \bar{p} \in (2/x, 1]$	$\bar{p} \in [0, \bar{p}_{01}]$ $\wedge \bar{p} = 1$
range of $\bar{p}$ if $x \leq 2$	$\bar{p} \in [0, 1]$	PBE cannot be selected	$\bar{p} \in [0, \bar{p}_{02}]$ $\wedge \bar{p} = 1$

Table 3: Different Ranges from which the Agency Can Choose $\bar{p}$

setting $k \leq \min\{1/2, x - 3/2\}$ (see Lemma 3), which is equivalent to $\bar{p} \in [0, \min\{\bar{p}_{01}, \bar{p}_{02}\}]$, or it can allow this selection by setting $k > \min\{1/2, x - 3/2\}$ which results in $\bar{p} = 1$. Suppose alternatively that the firms select the perfect Bayesian equilibrium with a positive demand and no unrated firms financed in case of multiple equilibria. Then $\bar{p} \in [0, \min\{2/x, 1\}]$ if $k \leq \min\{g(2/x; x), x - 1\}$ and $\bar{p} = 1$ for $k > \min\{g(2/x; x), x - 1\}$. If $x > 2$ holds, the firms may also select the perfect Bayesian equilibrium with a positive demand and all firms financed in case of multiple equilibria. Then $k \leq 1$ generates $\bar{p} \in [0, \bar{p}_k]$, $1 < k \leq g(2/x; x)$ generates $\bar{p} \in [2/x, 1)$ and $k > g(2/x; x)$ generates $\bar{p} = 1$. However, from Lemma 4 it is already obvious that the rating agency can always do better in this case than choosing $1 < k < g(2/x; x)$ because the profit function is downward sloping in $\bar{p}$ for $\bar{p} \in (2/x, 1)$, since the willingness to pay for a rating as well as the demand for rating decreases. Thus, there will never be an equilibrium with a positive demand for rating and financed firms which are not rated.

Taking into account Lemma 4 and the respective equilibrium selection of the firms we arrive at Proposition 3 and 4 where we use the following definitions:

$$\bar{a}_1 = \bar{a}(2/x, x), \quad \bar{a}_{01} = \bar{a}(\bar{p}_{01}, x) \text{ and } \bar{a}_{02} = \bar{a}(\bar{p}_{02}, x) \tag{8}$$

$$\text{with } \bar{a}(\bar{p}, x) = \left\{ \frac{c}{x} \middle| \, \Pi(\bar{p}; x, c) = 0, \right\}$$

$$\hat{a} = \left\{ \frac{c}{x} \middle| \, \Pi(2/x; x, c) = \Pi(\bar{p}_k; x, c) \right\} \tag{9}$$

$$\text{and } \underline{a}_k = \underline{a}(\bar{p}_k, x) \quad \underline{a}_{01} = \underline{a}(\bar{p}_{01}, x) \text{ and } \underline{a}_{02} = \underline{a}(\bar{p}_{02}, x) \tag{10}$$

where $\underline{a}(\bar{p}, x)$ is defined in (5). These definitions relate c/x to $1/x$ in order to be able to describe all possible perfect Bayesian equilibria for all possible combinations of the parameters $c < x$ and $x > 1$.

Proposition 3 *If firms select a perfect Bayesian equilibrium with a positive demand in case of multiple equilibria and* $c/x < \min\{\bar{a}_1, 1-1/x\}$*, then the rating agency chooses the rating fee* k *such that there is a positive demand for its services and all the rated, but none of the unrated firms are financed. The demand in equilibrium is characterized in Table 4. If* $c/x \geq \min\{\bar{a}_1, 1-1/x\}$*, the rating agency cannot survive. The firms are not financed in this case, if* $x \leq 2$*, whereas all the firms are financed, if* $x > 2$ *holds.*

<table>
<tr><th>x</th><th>$\bar{p}$ if firms select PBE with $\bar{p} \leq 2/x$ with multiple PBE</th><th>$\bar{p}$ if firms select PBE with $\bar{p} > 2/x$ with multiple PBE</th><th>$\bar{p}$ if firms select PBE with $\bar{p} = 1$ with multiple PBE</th></tr>
<tr><td rowspan="3">$x > 2$</td><td>$\bar{p} = 1 \Leftrightarrow$
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$c/x \geq \bar{a}_1$</td><td>$\bar{p} = 1 \Leftrightarrow$
$c/x \geq \bar{a}_{01}$</td></tr>
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$\bar{p} = \bar{p}_k \Leftrightarrow$
$\hat{a} > c/x \geq \underline{a}_k$</td><td>$\bar{p} = \bar{p}_{01} \Leftrightarrow$
$\bar{a}_{01} > c/x \geq \underline{a}_{01}$</td></tr>
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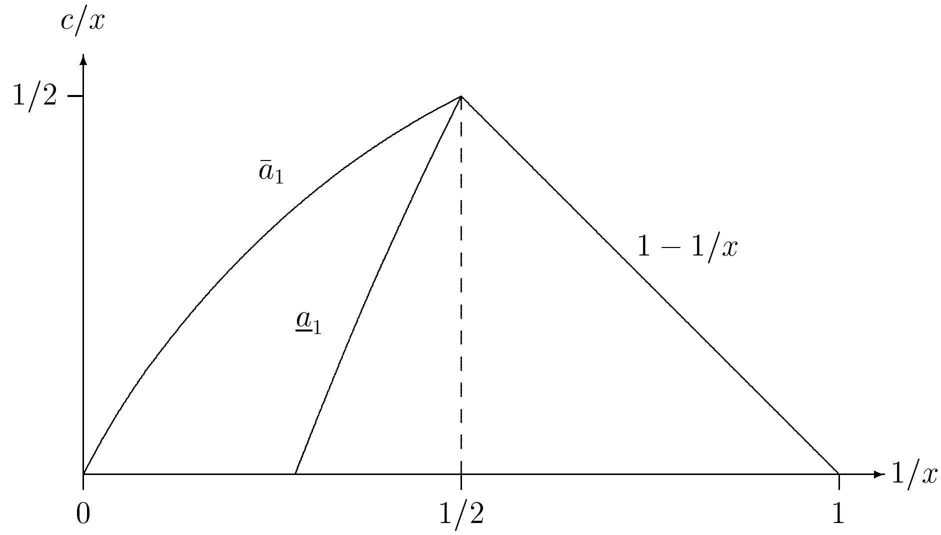
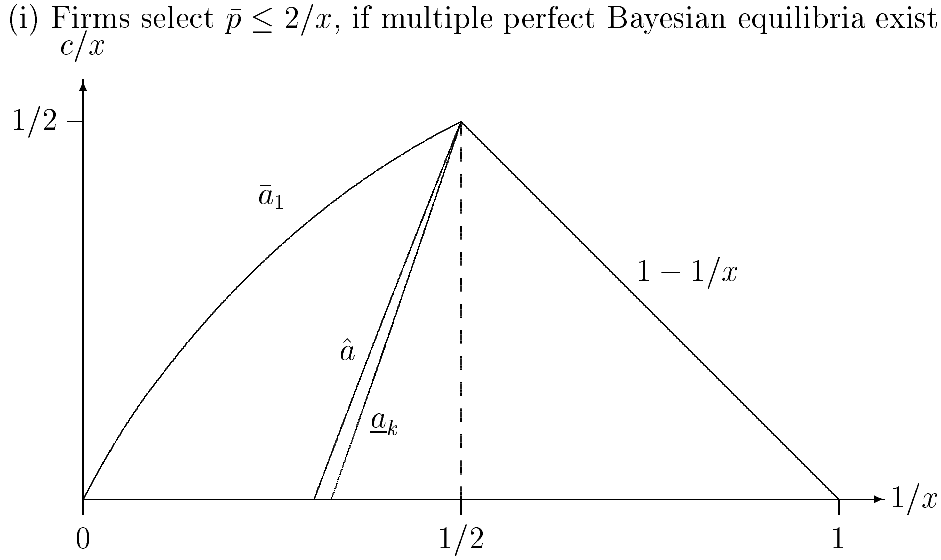
$\underline{a}_k > c/x > 0$</td><td>$\bar{p} \in (\max\{(3-x)$
$/(2x), 0\}, \bar{p}_{01}) \Leftrightarrow$
$\underline{a}_{01} > c/x > 0$</td></tr>
<tr><td rowspan="3">$x \leq 2$</td><td>$\bar{p} = 1 \Leftrightarrow$
$c/x \geq 1 - 1/x$</td><td rowspan="3">PBE cannot be selected</td><td>$\bar{p} = 1 \Leftrightarrow$
$c/x \geq \bar{a}_{02}$</td></tr>
<tr><td rowspan="2">$\bar{p} \in ((3-x)/(2x), 1) \Leftrightarrow$
$1 - 1/x > c/x > 0$</td><td>$\bar{p} = \bar{p}_{02} \Leftrightarrow$
$\bar{a}_{02} > c/x \geq \underline{a}_{02}$</td></tr>
<tr><td>$\bar{p} \in ((3-x)/(2x), \bar{p}_{02})$
$\Leftrightarrow \underline{a}_{02} > c/x < 0$</td></tr>
</table>

Table 4: Characterization of $\bar{p}$ in Equilibrium

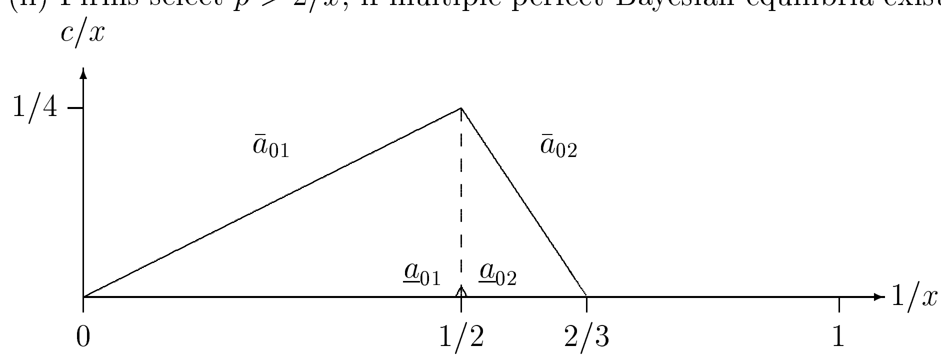
Proposition 3 is illustrated in Figure 2 (i) and (ii). If the firms select an equilibrium with a positive demand in the case of multiple perfect Bayesian equilibria existence of such an equilibrium requires that the rating costs c relative to the pay-off in case of success x should not exceed a certain threshold.



(i) Firms select $\bar{p} \leq 2/x$, if multiple perfect Bayesian equilibria exist



(ii) Firms select $\bar{p} > 2/x$, if multiple perfect Bayesian equilibria exist



(iii) Firms select $\bar{p} = 1$, if multiple perfect Bayesian equilibria exist

Figure 2: Existence of an Equilibrium with a Positive Demand

For $x \leq 2$ firms are not financed without the rating agency. Therefore a profit maximizing rating agency can survive in equilibrium as long as $x > c + 1$ meaning that the pay-off in case of success is sufficient to cover the rating costs and the risk-free interest rate factor. This is no longer true if $x > 2$. In this case an increase in x allows only a less than proportional increase in c because the maximum price $k = g(2/x; x)$ increases less than proportionally in x.

If the firms stick always to the perfect Bayesian equilibrium with $\bar{p} \leq 2/x$ ((i) in Figure 2), the rating agency chooses k such that the corner solution $\bar{p} = 2/x$ is induced as long as $\underline{a}_1 < c/x < \bar{a}_1$. For these combination of parameters the profit function of the rating agency has a single peak at this corner solution (see Lemma 4). Whereas for $c/x < \min\{\underline{a}_1, 1 - 1/x\}$ the rating agency chooses k such that $\bar{p} \in ((3 - x)/(2x), \min\{2/x, 1\})$ because the profit function has an interior maximum in this area. If to the contrary the firms select the perfect Bayesian equilibrium with $\bar{p} > 2/x$, if multiple perfect Bayesian equilibria exist, the area for corner solutions increases (see (ii) in Figure 2). The rating agency can never induce $\bar{p} \in (\bar{p}_k, 2/x)$, no matter whether the profit function has a maximum there or not, but it can either ensure $\bar{p} = \bar{p}_k$ or $\bar{p} = 2/x$. The former is the more profitable strategy if $\underline{a}_k < c/x < \hat{a}$. The latter is preferred if $\hat{a} < c/x < \bar{a}_1$. For $c/x < \min\{\underline{a}_k, 1 - 1/x\}$ the rating agency chooses k such that $\bar{p} \in ((3 - x)/(2x), \min\{\bar{p}_k, 1\})$ because the interior maximum of the profit function lies in this area.

Proposition 4 *If firms select a perfect Bayesian equilibrium without any demand in case of multiple equilibria and $c/x < \min\{\bar{a}_{01}, \bar{a}_{02}\}$, then the rating agency chooses the rating fee k nevertheless such that a positive demand is ensured. The unrated firms are never financed and for $0.64 < 1/x < 2/3$ this is true also for the firms that are rated "low". The demand in equilibrium is characterized in Table 4. If $c/x \geq \min\{\bar{a}_{01}, \bar{a}_{02}\}$ the rating agency cannot survive. No firms are financed in this case, if $x \leq 2$, whereas all the firms are financed, if $x > 2$ holds.*

If firms select the perfect Bayesian equilibrium without any demand, as soon as it exists, not surprisingly, there are much less combinations of c/x and $1/x$ for which an equilibrium with a positive demand exists (see Figure 2(iii)). With the exception of a small area with $c/x < \min\{\underline{a}_{01}, \underline{a}_{02}\}$ where $\bar{p} \in ((3 - x)/(2x), \min\{\bar{p}_{01}, \bar{p}_{02}\})$ is realized the rating agency chooses k such that the respective corner solution $\bar{p} = \bar{p}_{01}$ or $\bar{p} = \bar{p}_{02}$ is induced, meaning that the existence of the equilibrium without any demand is just prevented. For $0.64 < 1/x < 2/3$ the necessary prices are even that low that demand is driven from Case II to Case I illustrated in Figure 1, meaning that only the "high" rated firms are financed.

5 The Welfare Maximizing Supply of Rating Services

Now we want to compare the outcome of the game between the firms, the lenders and the monopolistic rating agency with the optimal supply of rating services from a social planners point of view. If a social planner were able to determine $\bar{p}$, he would maximize the following social welfare function W:

$$W = \begin{cases} -\int_{\bar{p}}^{1} cdp \text{ for } 0 \leq \bar{p} < (3 - 2x + \sqrt{3(3 + 4x - 4x^2)})/(4x), \\ \int_{\bar{p}}^{1} p^2 \left[x - \frac{1}{\rho_h(\bar{p})}\right] - cdp \text{ for } \max\{0, \\ (3 - 2x + \sqrt{3(3 + 4x - 4x^2)})/(4x)\} \leq \bar{p} < (3 - x)/(2x), \\ \int_{\bar{p}}^{1} p \left[x - \left(\frac{p}{\rho_h(\bar{p})} + \frac{1-p}{\rho_l(\bar{p})}\right)\right] - cdp \\ \text{for } \max\{0, (3 - x)/(2x)\} < \bar{p} \leq \min\{2/x, 1\}, \\ \int_{\bar{p}}^{1} p \left[x - \left(\frac{p}{\rho_h(\bar{p})} + \frac{1-p}{\rho_l(\bar{p})}\right)\right] - cdp + \int_0^{\bar{p}} p \left(x - \frac{1}{\rho_n(\bar{p})}\right) dp \\ \text{for } 2/x < \bar{p} \leq 1. \end{cases} \tag{11}$$

It results from the fact that, if $0 \leq \bar{p} < (3 - 2x + \sqrt{3(3 + 4x - 4x^2)})/(4x)$ no rated firm obtains a credit. If $\max\{0, (3 - 2x + \sqrt{3(3 + 4x - 4x^2)})/(4x)\} \leq \bar{p} < (3 - x)/(2x)$, only the "high" rated firms, can finance their investment project and generate a rent, whereas with $\max\{(3 - x)/(2x), 0\} \leq \bar{p} \leq \min\{2/x, 1\}$ this is true for all the rated firms. If $2/x < \bar{p} \leq 1$ all the firms obtain a credit and are able to generate a rent from their investment project. From the analysis of (11) we can derive the following Proposition:

Proposition 5 *For $x > 2$ the social planner would choose $\bar{p} = c/x + 1/x$ as long as $c/x \leq 1 - 1/x - \sqrt{1 - 2/x}$ and would prefer no supply of rating services ($\bar{p} = 1$) for $c/x > 1 - 1/x - \sqrt{1 - 2/x}$. For $x \leq 2$ the social planner would choose $\bar{p} = c/x + 1/x$ as long as $c/x < 1 - 1/x$ and no supply of rating services for $c/x \geq 1 - 1/x$.*

The social planner prefers that no firm demands a rating, if c/x exceeds a certain threshold. The latter is lower for $x > 2$ than for $x \leq 2$. In the former case all the firms would receive a credit without any rating, and the positive demand for rating limits the financing to those whose expected pay-off covers the rating cost and the risk-free interest rate factor. In the latter case, however, the rating is essential for the firms to obtain a credit and to be able to invest. The comparison of the social planner's preferred demand levels for rating with the ones that result from the game between the firms, the lenders and the monopolistic rating agency leads to the following proposition:

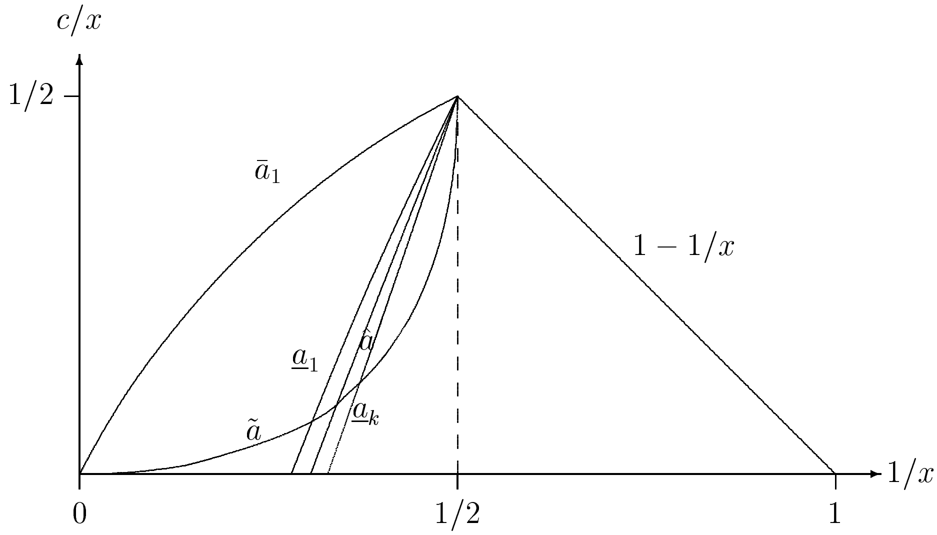


Figure 3: Oversupply and Undersupply of Rating Services under the Conditions of Proposition 6

Proposition 6 *If the firms select one of the two perfect Bayesian equilibria with a positive demand in case of multiple perfect Bayesian equilibria, the monopolistic rating agency supplies less of its services than the social planner would prefer if $x > 2$ and $c/x \leq 1 - 1/x - \sqrt{1 - 2/x} \equiv \tilde{a}$, whereas the rating agency supplies more, if $x > 2$ and $\tilde{a} < c/x \leq \bar{a}_1$. For $x \leq 2$ the monopolistic rating agency supplies always less in the equilibrium with a positive demand than the social planner would prefer.*

Proposition 6 is illustrated in Figure 3. The undersupply of rating services for $c/x \leq \tilde{a}$ can be explained with the usual tendency of monopolies to reduce their supply in order to maximize profits. The oversupply that can only occur, if $x > 2$ and $\tilde{a} < c/x \leq \bar{a}$ is due to an externality. If some firms decide in favor of a rating, others have to follow them in order not to be grouped together with the bad risks. This phenomenon is always present, but is valued differently by the social planner. For $x > 2$ the unrated firms would obtain a credit without the rating, but lose it when the rating agency is present. These losses and the rating costs cannot be compensated by the gains of the rated firms which pay a lower interest rate factor. For $x \leq 2$ no firm would obtain a credit without the rating agency. Thus, the rating and the resulting credit is always positive from the social planner's point of view and has to cover just its costs. Here the monopolist's tendency to undersupply always dominates.

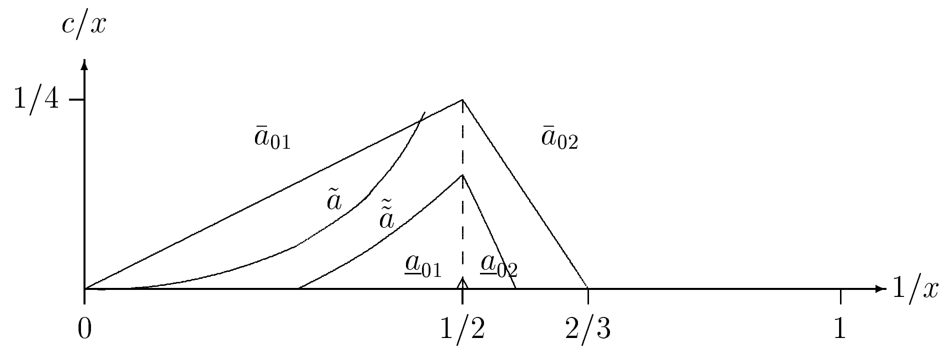


Figure 4: Oversupply and Undersupply of Rating Services under the Conditions of Proposition 7

Proposition 7 *If the firms select the perfect Bayesian equilibrium without any rating demand in case of multiple perfect Bayesian equilibria, the monopolistic rating agency supplies less of its services than the social planner would prefer if* $c/x \leq \min\{\bar{p}_{01}, \bar{p}_{02}\} - 1/x \equiv \tilde{\tilde{a}}$. *It supplies more, if* $\min\{\bar{a}_{01}, \bar{a}_{02}\} > c/x > \tilde{\tilde{a}}$.

Proposition 7 is illustrated in Figure 4. Note that if the firms select the perfect Bayesian equilibrium without any demand as soon as it exists, oversupply cannot only occur if $x > 2$ but also if $x \leq 2$ and $\min\{\bar{a}_{01}, \bar{a}_{02}\} > c/x > \tilde{\tilde{a}}$. This is due to the fact that the rating agency must stick to a rather low rating price in order to prevent the existence of the perfect Bayesian equilibria without any rating demand. At this price demand exceeds the level preferred by the social planner if the rating cost relative to the pay-off in case of success exceeds $\tilde{\tilde{a}}$.

6 Conclusions

We have shown in this paper that the demand for rating services has some surprising properties, if the pay-off in case of success is high enough that the firms would also obtain a credit without a rating agency. The willingness to pay for a rating is increasing for low demands and decreasing for higher demands. The reason is that for low demands the unrated firms still obtain a credit and their credit risk would be considered to be relatively low. Therefore the low risks only demand a rating, if its price is pretty low. If the rating demand increases, the assumed credit risk of the unrated increases and the willingness to pay for the rating rises, until the unrated do not receive a credit anymore. At this point further demand for the rating can only be produced with a lowering of the rating price.

The rating agency, however, always supplies its services at the decreasing part of the inverse demand curve, because its marginal profits at the increasing part is negative. Thus, unrated firms are never financed by the capital market, if a rating agency serves any demand for rating. This is true no matter whether unrated firms obtained a credit or not if no rating agency existed.

Given that the firms select a perfect Bayesian equilibrium with a positive demand, if multiple perfect Bayesian equilibria exist, there is always an undersupply of rating services from a social planners point of view, if no firm would have obtained a credit without the rating agency. Thus, here the usual undersupply tendency of a monopolist dominates the outcome of the game. If, however, all firms would have obtained a credit without the rating agency, there can be oversupply. This is due to the fact that those that do not demand a rating are not financed any more and lose their rents. The gains of the rated via the reduced interest rate factor may not compensate for these losses and the rating costs. Thus, in the latter case some kind of a herding effect dominates the outcome of the game, because if there is any positive demand for rating this creates the incentive for other firms to demand a rating as well in order not to be grouped together with the bad risks.

If the firms select a perfect Bayesian equilibrium without any demand as soon as it exists, there can also be oversupply from a social welfare point of view even if no firm obtained a credit without the rating agency. The reason for this is the low rating fee that the rating agency must set in order to prevent the existence of this equilibrium. Thus, oversupply is here due to the rating agency's strategic reply to the equilibrium selection of the firms.

Contrary to the general perception of an information generating rating agency in a market that suffers from adverse selection phenomena, we showed with our model that there might be too much of a good thing. The model is, however, rather specific, and it would be interesting to find out whether its main results would be maintained in a more general framework. It would be especially worthwhile to consider competition among rating agencies, because the market for rating services in reality seems to be characterized more by an oligopolistic market structure than by a monopoly.

Appendix

Proof of Lemma 1

A firm demands a rating, given the lenders' beliefs about their success probabilities, if the following inequality holds (see (2)):

$$\begin{aligned} p \max\left\{p\left(x-\frac{1}{\rho_h}\right),0\right\} + (1-p)\max\left\{p\left(x-\frac{1}{\rho_l}\right),0\right\} - k \\ > \max\left\{p\left(x-\frac{1}{\rho_n}\right),0\right\} \end{aligned} \tag{12}$$

Suppose $\rho_h > \rho_l > \rho_n$, then all the firms with a success probability that satisfies:

$$p > \bar{p} \equiv \begin{cases} \frac{\sqrt{\rho_h\left[\rho_h(\rho_l-\rho_n)^2+4k(\rho_h-\rho_l)\rho_l(\rho_n)^2\right]}-\rho_h(\rho_l-\rho_n)}{2\rho_n(\rho_h-\rho_l)} \text{ for } x > \frac{1}{\rho_n}, \\ \frac{\sqrt{\rho_h[\rho_h(\rho_l x-1)^2+4k(\rho_h-\rho_l)\rho_l]}-\rho_h(\rho_l x-1)}{2(\rho_h-\rho_l)} \text{ for } \frac{1}{\rho_n} \geq x \geq \frac{1}{\rho_l}, \\ \frac{\sqrt{k\rho_h}}{\sqrt{\rho_h x-1}} \text{ for } \frac{1}{\rho_l} > x \geq \frac{1}{\rho_h}. \end{cases} \tag{13}$$

demand a rating, given $\bar{p} < 1$.[12] The beliefs of the lenders according to Bayes' rule should be:

$$\rho_h(\bar{p}) = \frac{\int_{\bar{p}}^1 p^2 dp}{\int_{\bar{p}}^1 p dp} = \frac{2(1+\bar{p}+\bar{p}^2)}{3(1+\bar{p})}, \tag{14}$$

$$\rho_l(\bar{p}) = \frac{\int_{\bar{p}}^1 p(1-p)dp}{\int_{\bar{p}}^1 (1-p)dp} = \frac{1+2\bar{p}}{3}, \tag{15}$$

$$\rho_n(\bar{p}) = \frac{\int_0^{\bar{p}} p dp}{\int_0^{\bar{p}} 1 dp} = \frac{\bar{p}}{2}. \tag{16}$$

Thus, lenders beliefs and firms behavior are consistent, because $\rho_h(\bar{p}) > \rho_l(\bar{p}) > \rho_n(\bar{p})$ holds for $\bar{p} < 1$. A separation of firms where all those with $p \leq \bar{p}$ do not and those with $\bar{p} < p < 1$ do demand a rating forms a perfect Bayesian equilibrium.

For all possible alternative assumptions concerning the rank order of ρ_h, ρ_l and ρ_n we can also derive the demand for rating services. The beliefs that are formed according to Bayes rule are, however, never consistent with the underlying assumption about the rank order. Thus, all possible perfect Bayesian equilibria with a positive demand must be characterized by (13).

[12] If $x < 1/\rho_h$ holds, the lenders would not even finance "high" rated firms. Thus, no firm demands a rating.

Proof of Proposition 2 and Lemma 2

For $x > 2$ the function $g(\bar{p};x)$ has a maximum at $\bar{p} = 2/x$, because the following inequalities hold:

$$\frac{\partial g(\bar{p};x)}{\partial \bar{p}} = \begin{cases} \bar{p}\left(2x - \frac{3(2+4\bar{p}+2\bar{p}^2+\bar{p}^3)}{2(1+\bar{p}+\bar{p}^2)^2)}\right) \geq 0 & \text{for all } 0 \leq \bar{p} < \frac{3-x}{2x} \\ \left(x - \frac{3(2+2\bar{p}+6\bar{p}^2+10\bar{p}^3+7\bar{p}^4)}{2(1+3\bar{p}+3\bar{p}^2+2\bar{p}^3)^2}\right) \geq 0 & \text{for all } \max\left\{\frac{3-x}{2x},0\right\} \leq \bar{p} < \min\left\{\frac{2}{x},1\right\} \\ -\frac{3(2+2\bar{p}+6\bar{p}^2+10\bar{p}^3+7\bar{p}^4)}{2(1+3\bar{p}+3\bar{p}^2+2\bar{p}^3)^2} < 0 & \text{for all } \frac{2}{x} < \bar{p} < 1 \end{cases}$$

Thus, a perfect Bayesian equilibrium with a positive demand for the rating services exists for a sufficiently low but positive rating fee, as long as:

$$g(2/x;x) > 0 \Leftrightarrow \frac{2\left(x^3+3x^2+9x-2\right)}{(x^3+6x^2+12x+16)} > 0 \Leftrightarrow x > 2.$$

The perfect Bayesian equilibrium with a positive demand for a given rating price is unique for $0 < k \leq g(1,x)$ and implies that only rated firms are financed. If $g(2/x,x) > k > g(1,x) = 1$ holds, then there are two perfect Bayesian equilibria with $\bar{p} \in (0,1)$. In one equilibrium $\bar{p} < 2/x$ and the unrated firms are not financed by the capital market. In the other equilibrium $\bar{p} > 2/x$ and the unrated firms are financed. In the limit case of $k = g(2/x;x)$ two perfect Bayesian equilibria exist, in both $\bar{p} = 2/x$ holds but in one the unrated firms are financed and in the other one they are not financed. For any rating fee that implies $k > g(2/x;x)$ no perfect Bayesian equilibrium with a positive demand exists.

For $x \leq 2$, the function $g(\bar{p};x)$ is monotonously increasing in $\bar{p}$, because:

$$\frac{\partial g(\bar{p};x)}{\partial \bar{p}} = \begin{cases} 0 \text{ for all } 0 \leq \bar{p} < \frac{3-2x+\sqrt{3(3+4x-4x^2)}}{4x} \\ \bar{p}\left(2x - \frac{3(2+4\bar{p}+2\bar{p}^2+\bar{p}^3)}{2(1+\bar{p}+\bar{p}^2)^2)}\right) \geq 0 \\ \text{for all } \max\left\{\frac{3-2x+\sqrt{3(3+4x-4x^2)}}{4x},0\right\} \leq \bar{p} < \frac{3-x}{2x} \\ \left(x - \frac{3(2+2\bar{p}+6\bar{p}^2+10\bar{p}^3+7\bar{p}^4)}{2(1+3\bar{p}+3\bar{p}^2+2\bar{p}^3)^2}\right) \geq 0 \text{ for all } \frac{3-x}{2x} \leq \bar{p} < 1 \end{cases} \tag{17}$$

Thus, a unique perfect Bayesian equilibrium with a positive demand exists for all k with $0 < k < g(1;x) = x-1$. Since $\bar{p} < 2/x$ for all $\bar{p} \in [0,1)$, only rated firms are financed. If $k \geq x-1$ no perfect Bayesian equilibrium with a positive rating demand exists.

Proof of Lemma 3

Given Assumption 1, the lenders' out-of-equilibrium beliefs about a firm's success probability that is rated "high" ("low") should coincide with $\rho_h(\tilde{p})$ from (14) ($\rho_l(\tilde{p})$ from (15)). Both beliefs increase in the threshold $\tilde{p}$. Thus, the most pessimistic out-of-equilibrium belief the lenders can possibly hold about the success probability of a rated firm and, thus, the most favorable belief for the existence of a perfect Bayesian equilibrium without any rating demand is represented by:

$$\lim_{\tilde{p}\searrow 0} \rho_h(\tilde{p}) = \frac{2}{3} \text{ and } \lim_{\tilde{p}\searrow 0} \rho_l(\tilde{p}) = \frac{1}{3}.$$

Given these beliefs, all the firms would refrain from rating, if:

$$p \max\left\{p\left(x - \frac{3}{2}\right), 0\right\} + (1-p)\max\left\{p(x-3), 0\right\} - k < \max\left\{p(x-2), 0\right\}$$

for all $p \in [0, 1)$. For $x > 2$, this is equivalent to $k > 1/2$. For $x \leq 2$ this implies $k > x - 3/2$.

Proof of Lemma 4

Note first that the rating agency's profit function is continuous in $\bar{p}$. Note also that $g(\bar{p}, x) = 0$ holds for all $\bar{p} \in \left[0, (3 - 2x + \sqrt{3(3 + 4x - 4x^2)})/(4x)\right)$ (see (3)). Thus, the agency's profit trivially increases in $\bar{p} \in \left[0, (3 - 2x + \sqrt{3(3 + 4x - 4x^2)})\right.$ $/(4x)]$ (see (4)).
In addition we can show:

$$\frac{\partial \Pi\left(\max\{(3 - 2x + \sqrt{3(3 + 4x - 4x^2)})/(4x), 0\}; x, c\right)}{\partial \bar{p}} \geq 0, \tag{18}$$

$$\frac{\partial^2 \Pi\left(\max\{(3 - 2x + \sqrt{3(3 + 4x - 4x^2)})/(4x), 0\}; x, c\right)}{\partial \bar{p}^2} \geq 0, \tag{19}$$

$$\lim_{\tilde{p}\rightarrow \frac{3-x}{2x}} \frac{\partial \Pi(\bar{p}; x, c)}{\partial \bar{p}} \geq 0. \tag{20}$$

Given (18), (19) and (20), the function $\Pi(\bar{p}; x, c)$ can only decrease in $\bar{p}$ for $\bar{p} \in \left[\max\{(3 - 2x + \sqrt{3(3 + 4x - 4x^2)})/(4x), 0\}, (3 - x)/(2x)\right)$, if there are at least two values of $\bar{p} \in \left[\max\{(3 - 2x + \sqrt{3(3 + 4x - 4x^2)})/(4x), 0\}, (3 - x)/(2x)\right)$ for which $\partial^2 \Pi(\bar{p}; 1/x, c/x)/(\partial \bar{p}^2) = 0$ holds. The latter means that the rating agency's profit function must change from a convex into a concave and back into a convex function. It can be shown, however, that $\partial^2\Pi(\bar{p};x,c)/(\partial\bar{p}^2) = 0$ holds at most for one value of $\bar{p}$ where the function turns from a convex into a concave function. Thus, $\Pi(\bar{p};x,c)$ is monotonously increasing in $\bar{p}$ for $\bar{p} \in \big[\max\{(3-2x+\sqrt{3(3+4x-4x^2)})/(4x), 0\}, (3-x)/(2x)\big)$.

Similar arguments apply for $\bar{p} \in [\max\{0,(3-x)/(2x)\}, \min\{2/x,1\})$ where

$$\frac{\partial\Pi(\max\{0,(3-x)/(2x)\};x,c)}{\partial\bar{p}} \geq 0 \text{ and} \tag{21}$$

$$\lim_{\bar{p}\to\min\{1/(2x),1\}} \frac{\partial\Pi(\bar{p};x,c)}{\partial\bar{p}} \geq 0 \Leftrightarrow \frac{c}{x} \geq \min\{\underline{a}_1, 1-1/x\} \tag{22}$$

$$\begin{aligned} \text{with } \underline{a}_1 \equiv & \big[448(1/x)^7 + 416(1/x)^6 + 504(1/x)^5 + 272(1/x)^4 + 46(1/x)^3 \\ & -18(1/x)^2 - 5(1/x) - 1\big] \cdot \frac{1}{(16(1/x)^3 + 12(1/x)^2 + 6(1/x) + 1)^2} \end{aligned} \tag{23}$$

hold. If $c/x < \min\{\underline{a}_1, 1-1/x\}$ the function $\Pi(\bar{p};x,c)$ obviously has a maximum at $\bar{p}^*$ with $\bar{p}^* \in [\max\{0,(3-x)/(2x)\}, \min\{2/x,1\})$. The value $\bar{p}^*$ can be uniquely determined by solving $(\partial\Pi(\bar{p};x,c))/(\partial\bar{p}) = 0$ for $\bar{p}$, because $(\partial^2\Pi(\bar{p};x,c))/(\partial\bar{p}^2) = 0$ holds at most for one value of $\bar{p} \in [\max\{0,(3-x)/(2x)\}, \min\{2/x,1\})$, where $\Pi(\bar{p};x,c)$ turns from a convex into a concave function. Thus, $\Pi(\bar{p};x,c)$ is either monotonously increasing in $\bar{p}$ for $\bar{p} \in [0, \min\{2/x,1\})$, if (22) holds or has a maximum at $\bar{p}^*$ with $\bar{p}^* \in [\max\{0,(3-x)/(2x)\}, \min\{2/x,1\})$.

The rating agency's profit function decreases in $\bar{p}$ for all $\bar{p} \in (2/x, 1)$ because $\partial g(\bar{p};x)/\partial\bar{p} < 0$ from (17).

Proof of Proposition 3 and Proposition 4

Suppose that firms select the perfect Bayesian equilibrium with $\bar{p} \leq 2/x$ if multiple perfect Bayesian equilibria exist and that $\bar{p}^* = 2/x$. Then the rating agency induces $\bar{p}^*$ as long as $\Pi(2/x;x,c) > 0$ which is equivalent to:

$$g\left(\frac{2}{x};x\right) < c \Leftrightarrow \frac{c}{x} < \frac{2/x(1+3/x+9/x^2-2/x^3)}{1+6/x+12/x^2+16/x^3} \equiv \bar{a}_1. \tag{24}$$

and $\bar{p} = 1$ otherwise. Suppose that firms select the perfect Bayesian equilibrium with $\bar{p} \leq 2/x$ if multiple perfect Bayesian equilibria exist and, alternatively, that $\bar{p}^* \in [\max\{0,(3-x)/(2x)\}, \min\{2/x,1\})$. This means that $\Pi(\bar{p}^*;x,c) > \Pi(\min\{2/x,1\};x,c)$. Given $x > 2$, $\Pi(\bar{p}^*;x,c) > \Pi(2/x;x,c) > 0$ is ensured as long as (24) holds. Given $x < 2$, $\Pi(\bar{p}^*;x,c) > \Pi(1;x,c) = 0$ holds. Thus, if $\bar{p}^* < 1 \Leftrightarrow c/x < 1-1/x$, then the rating agency's profit is always positive.

Suppose now that $x > 2$ and that firms select the perfect Bayesian equilibrium with $\bar{p} > 2/x$ if multiple perfect Bayesian equilibria exist. Then the rating agency induces $\bar{p}^* = 2/x$ as long as $\Pi(2/x; x, c) > \max\{0, \Pi(\bar{p}_k; x, c)\}$ which is equivalent to $\bar{a}_1 > c/x > \hat{a}$. It induces $\bar{p} = \bar{p}_k > (3-x)/(2x)$ as long as its profit function is monotonously increasing for $\bar{p} \in [\max\{0, (3-x)/(2x)\}, \bar{p}_k)$ and $\Pi(\bar{p}_k; x, c) > \Pi(2/x; x, c)$ which is equivalent to $\hat{a} > c/x \geq \underline{a}_k$. Since $\hat{a} < \bar{a}_1$ holds for all $x > 2$, $\Pi(\bar{p}_k; x, c) > \Pi(2/x; x, c) > 0$ is always satisfied. If $\bar{p}^* \in [\max\{0, (3-x)/(2x)\}, \bar{p}_k) \Leftrightarrow \underline{a}_k > c/x$ the rating agency sets $\bar{p}^*$, because a positive profit is ensured by $c/x < \underline{a}_k < \bar{a}_1 \Leftrightarrow \Pi(\bar{p}^*; x, c) > \Pi(2/x; x, c) > 0$.

Suppose now that firms select the perfect Bayesian equilibrium without any demand as soon as it exists. Then it induces $\bar{p}_{01} > (3-x)/(2x)$ for $x > 2$ as long as its profit function is monotonously increasing in $\bar{p}$ for $\bar{p} \in [(3-x)/(2x), \bar{p}_{01})$ and $\Pi(\bar{p}_{01}; x, c) > 0$ holds. This is equivalent to $\bar{a}_{01} > c/x > \underline{a}_{01}$. It ensures $\bar{p}^* \in [(3-x)/(2x), \bar{p}_{01})$ if $c/x < \underline{a}_{01}$ which implies $\Pi(\bar{p}^*; x, c) > 0$. It induces $\bar{p}_{02}$ for $x < 2$ as long as its profit function is monotonously increasing in $\bar{p}$ for $\bar{p} \in [0, \bar{p}_{02})$ and $\Pi(\bar{p}_{02}; x, c) > 0$. This is equivalent to $\bar{a}_{02} > c/x > \underline{a}_{02}$. Note that $\bar{p}_{02} < (3-x)/(2x)$ holds for $1/x > 0.64$ meaning that only the "high" rated firms obtain a credit. The rating agency ensures $\bar{p}^* \in [(3-x)/(2x), \bar{p}_{02})$ if $c/x < \underline{a}_{02}$ which implies $\Pi(\bar{p}^*; x, c) > 0$.

Proof of Proposition 5

Social welfare can be expressed as (see (11)):

$$W(\bar{p}; x, c) = \begin{cases} -c(1-\bar{p}) \text{ for } 0 \leq \bar{p} < \frac{3-2x+\sqrt{3(3+4x-4x^2)}}{4x}, \\ \frac{1}{6}\left[2x(1-\bar{p}^3) - 6c(1-\bar{p}) - 3(1-\bar{p}^2)\right] \text{ for } \max\{0, \\ \frac{3-2x+\sqrt{3(3+4x-4x^2)}}{4x}\} \leq \bar{p} < (3-x)/(2x) \\ (1-\bar{p})\left[\frac{(1+\bar{p})x}{2} - 1 - c\right] \text{ for } \max\{0, (3-x)/(2x)\} \\ < \bar{p} \leq \min\{2/x, 1\}, \\ \frac{x}{2} - 1 - c(1-\bar{p}) \text{ for } 2/x < \bar{p} \leq 1 \end{cases} \tag{25}$$

$W(\bar{p}; x, c)$ is continuous in $\bar{p}$. It is monotonously increasing for all $\bar{p}$ with $0 \leq \bar{p} < (3-x)/(2x)$. It has an interior maximum if $c < 1$ at $\bar{p} = c/x + 1/x$ in the interval $(\max\{0, (3-x)/(2x)\}, \min\{2/x, 1\}]$ and is monotonously increasing if $c > 1$. For $2/x < \bar{p} \leq 1$ the social welfare is again monotonously increasing. Thus, the social planner would prefer $\bar{p} = 1$ if:

$$W(1; x, c) > W(c/x + 1/x; x, c) \Leftrightarrow \frac{c}{x} > \begin{cases} 1 - \frac{1}{x} - \sqrt{1 - \frac{2}{x}} \text{ for } x > 2, \\ 1 - \frac{1}{x} \text{ for } x \leq 2 \end{cases}$$

and $\bar{p} = c/x + 1/x$ otherwise.

Proof of Proposition 6 and Proposition 7

If firms select one of the possibly two perfect Bayesian equilibria with a positive demand, the rating agency induces $\bar{p} \in (0,1)$ (see Proposition 3), as long as $c/x \leq \bar{a}_1$ for $x > 2$ and as long as $c/x < 1 - 1/x$ for $x < 2$. Since $\bar{a}_1 \geq 1 - 1/x - \sqrt{1 - 2/x}$ for $x > 2$, the rating agency supplies more services as the social planner would prefer if $\bar{a}_1 \geq c/x > 1 - 1/x - \sqrt{1 - 2/x}$ (see Proposition 5). If $c/x \leq 1 - 1/x - \sqrt{1 - 2/x}$ for $x > 2$ or if $c/x \leq 1 - 1/x$ for $x \leq 2$ the social planner would choose $\bar{p} = c/x + 1/x$. The rating agency, however, may either choose k such that it induces the corner solutions $\bar{p} = 2/x$ or $\bar{p} = \bar{p}_k$ or a $\bar{p}$ that satisfies (**??**) with equality. Taking into account the relevant ranges of c/x we can show that $2/x > 1/x + c/x$ and that $\bar{p}_k > 1/x + c/x$ holds. In addition $\partial \Pi(c/x + 1/x; x, c)/\partial \bar{p} > 0$ holds for those values of c/x for which the rating agency chooses a $\bar{p}$ that solves $\partial \Pi(\bar{p}; x, c)/\partial \bar{p} = 0$. Thus, given Lemma 4, $\bar{p} > c/x + 1/x$ holds.

If firms select the perfect Bayesian equilibrium without any rating demand, as soon as it exists, the same types of arguments go through for $\bar{a}_0 > c/x > 1 - 1/x - \sqrt{1 - 2/x}$ and are thus not repeated here. In addition $\bar{p}_{01} < c/x + 1/x$ and $\bar{p}_{02} < c/x + 1/x$ holds for $\min\{\bar{a}_{01}, \bar{a}_{02}\} \geq c/x > \min\{p_{01}, p_{02}\} - 1/x$ meaning oversupply. Using the same arguments as above it undersupplies for $c/x < \min\{p_{01}, p_{02}\} - 1/x$.

References

[1] **Albano, Gian L. and Lizzeri, Alessandro (1997):** Provision of Quality and Certification Intermediaries, Nota di Lavoro 37.97, Fondazione Eni Enrico Mattei, Milan.

[2] **Akerlof, George A. (1970):** The Market for Lemons: Quality and the Market Mechanism, *Quarterly Journal of Econommics*, Volume 84, pp. 488-500.

[3] **Biglaiser, Gary (1993):** Middleman as Experts, *Rand Journal of Economics*, Volume 24, pp. 212-223.

[4] **Fight, Andrew (2001):** The Ratings Game, Wiley, Chichester.

[5] **Kuhner, Christoph (2001):** Rating Agencies: Are they Credible? - Insights into the Reporting Incentives of Rating Agencies in Times of Enhanced Systemic Risk -, *Schmalenbach Business Review (ZfbF)*, Volume 53, pp. 2-26.

[6] **Leland, Hayne E. and Pyle, David H. (1977):** Informational Asymmetries, Financial Structure, and Financial Intermediation. *The Journal of Finance*, Volume 32, pp. 371-387.

[7] **Lizzeri, Alessandro (1999):** Information Revelation and Certification Intermediaries, *Rand Journal of Economics*, Volume 30, pp. 214-231.

[8] **Melumad, Nahum D. and Thoman, Lynda (1990):** On Auditors and the Courts in an Adverse Selection Setting, *Journal of Accounting Research*, Volume 28, pp. 77-120.

[9] **Milde, Hellmuth and Riley, John G. (1988):** Signalling in Credit Markets, *Quarterly Journal of Economics*, Volume 103, pp. 101-129.

[10] **Miller, Merton H. and Rock, Kevin (1985):** Dividend Policy under Asymmetric Information. *The Journal of Finance*, Volume 40, pp. 1031-1051.

[11] **Millon, Marcia H. and Thakor, Anjan V. (1985):** Moral Hazard and Information Sharing: A Model of Financial Information Gathering Agencies. *The Journal of Finance*, Volume 40, pp. 1403-1422.

[12] **Mukhopadhyay, Bappaditya (2000):** Rating Agencies - Efficiency and Regulations. Indian Statistical Institute, New Dehli, Mimeo.

[13] **Nayar, Nandkumar (1993):** Asymmetric Information, Voluntary Ratings and the Rating Agency of Malaysia, *Pacific-Basin Finance Journal*, Volume 1, pp. 369-380.

[14] **Ramakrishnan, Ram T. S. and Thakor, Anjan V. (1984):** Information Reliability and a Theory of Financial Intermediation. *Review of Economic Studies*, Volume 51, pp. 415-432.

[15] **Spence, Michael (1973):** Job Market Signalling, *Quarterly Journal of Economics*, Volume 87, pp. 355-374.